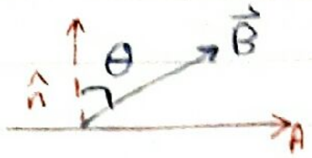
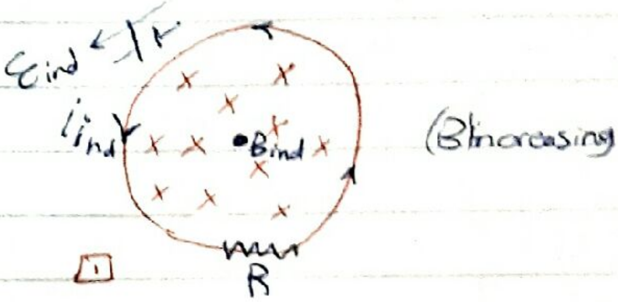
30 Induction and Inductance.

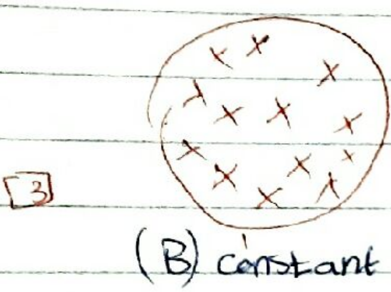
→ Magnatic Flux $\Phi_B = \int \vec{B} \cdot dA = \int B \cos\theta \, dA$ $T \cdot m^2$ "weber"

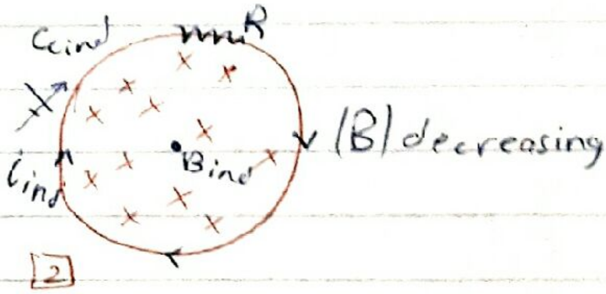


→ Farady's law:

$$\mathcal{E}_{ind} = -N \frac{d\Phi_B}{dt}$$





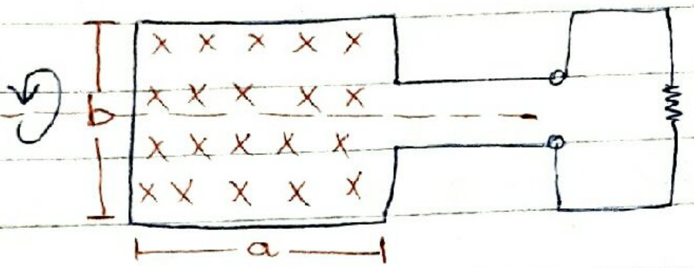


→ lenz law: The direction of ($\mathcal{E}_{ind}$ / i_{ind}) to oppeset the Change that product it.

→ ϕ_B Could be Changed by:

1. Changing B.
2. Changing A.
3. Changing θ.
4. Any Combination.

(30-31) Electric Generators:-



, Find ε_{ind} ??

$$\varepsilon_{ind} = (-)\, N \frac{d\Phi_B}{dt}$$

$$= (-)\, N \frac{d}{dt}(B a b \cos\theta) = (-)\, N B a b \frac{d \cos\theta}{dt}$$

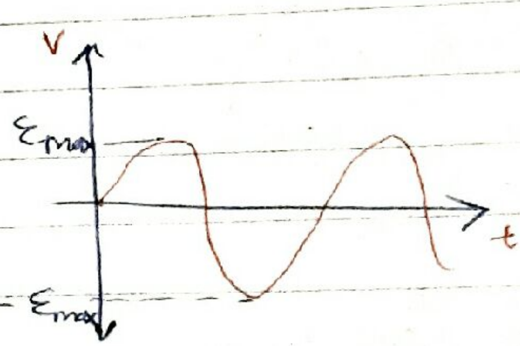
$$\varepsilon_{ind} = N B a b \sin\theta \frac{d\theta}{dt} \quad , \quad \omega = \frac{d\theta}{dt} = rad/s$$

$$\omega = 2\pi f$$

$$\varepsilon_{ind} = N B a b \omega \sin\theta$$

$$\varepsilon_{ind} = 2\pi f B a b \sin(\omega t)$$

$$\varepsilon_{ind} = \varepsilon_{max} \sin(\omega t)$$



⇒ Induction and Inductance:-

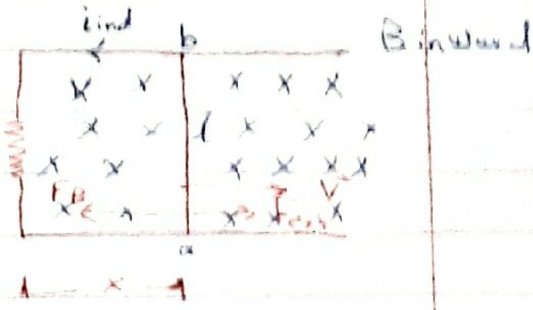
Farady's law → $\varepsilon_{ind} = (-)\, N \frac{d\Phi_B}{dt}$

$$\Phi_B = B \cdot A = B A \cos\theta$$

⇒ Inducting and Energy transfere:

$$\varepsilon_{ind} = (-) N \frac{d\phi_B}{dt}$$

$$= (-) \frac{d}{dt} (B l \cos 180)$$



$$\Rightarrow \varepsilon_{ind} = B l \frac{dx}{dt}$$

$$\Rightarrow B l V = \varepsilon_{ind} \text{ volt}$$

$$\Rightarrow i_{ind} = \frac{BlV}{R} \text{ Ampere}$$

$$\Rightarrow P_R = i^2 R = \frac{B^2 l^2 V^2}{R^2} . R = \frac{B^2 l^2 V^2}{R} \text{ watt}$$

$$\Rightarrow \vec{F_B} = i \vec{l} \times \vec{B} = \frac{BlV}{R} \times l \times B \sin 90 (-\hat{i})$$

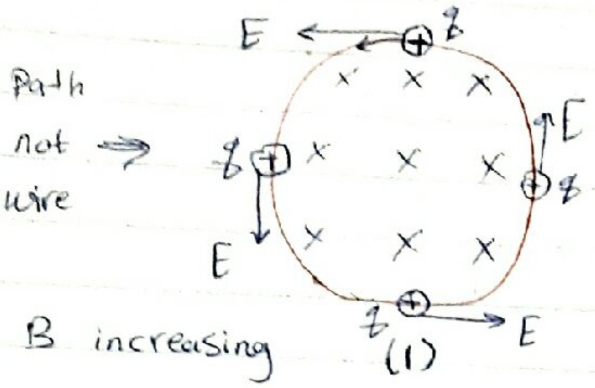
In the opposite direction

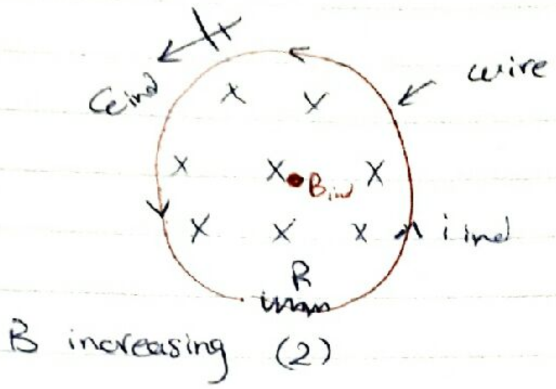
$$\Rightarrow \vec{F}_{ext} = \frac{B^2 l^2 V}{R} (\hat{i}) \text{ N}$$

$$\Rightarrow P_{F_{ext}} = \vec{F}_{ext} \cdot \vec{V} = \frac{B^2 l^2 V^2}{R}$$

$$\Rightarrow \boxed{P_{F_{ext}} = P_R}$$

⇒ Induced Electric field:





※ $\frac{dB}{dt}$ produce Induced electric field In a circular motion:

$$W_{\varepsilon ind} = q\,\varepsilon_{ind} \qquad (C.V) = J$$

$$\oint q\vec{E}\cdot ds = W_{\varepsilon ind}$$

$$q\,\varepsilon_{ind} = q\oint \vec{E}\cdot d\vec{s}$$

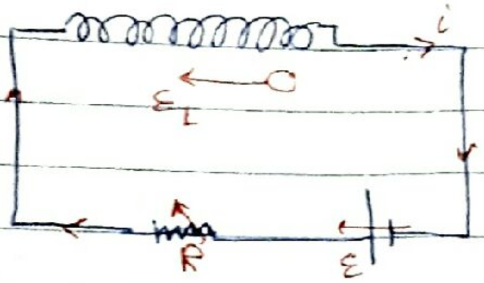
$$\varepsilon_{ind} = \oint \vec{E}_{ind}\cdot d\vec{s} = (-)\,N\,\frac{d\Phi_B}{dt}$$

⇒ Faraday's law:

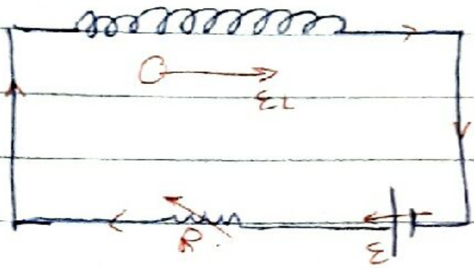
$$\oint \vec{E}_{ind}\cdot d\vec{s} = (-)\frac{d\Phi_B}{dt}$$

E_{ind} in a circular form is Non Conservative.

$\Rightarrow$ Inductance and Inductors :.



i increasing
↳ R ~~in~~ decreasing



i decreasing
↳ R ~~in~~ increasing

• Coil (inductor) has (inductance) $= \frac{N\Phi_B}{I}$

$$L = \frac{N\Phi_B}{I} \rightarrow \frac{\text{weber}}{\text{Amp}} = \text{Henry}$$

$$L I = N \Phi_B$$

$$L \frac{dI}{dt} = N \frac{d\Phi_B}{dt} = -\varepsilon_{ind}$$

$$\varepsilon_L = (-) N \frac{d\Phi_B}{dt} \quad \text{Volt}$$

Inductors has inductance (L)

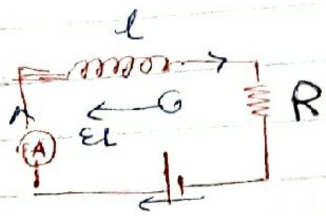
$$\rightarrow \mathcal{E}_L = (-)\, L \frac{di}{dt}$$

$$\rightarrow L = \frac{N \Phi_B}{I}$$

• L for Solenoid $= n^2 \mu_0 A l$

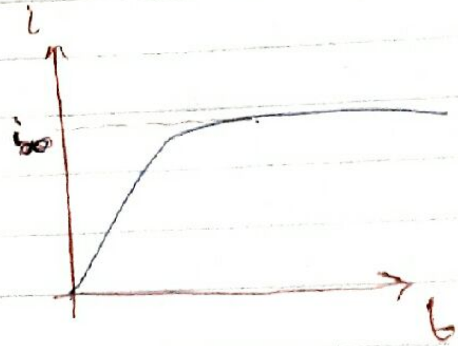
* L depends on Geometry.

⟹ RL - CirCuit :



• i increases gradually

form $i_0 = 0 \longrightarrow i_\infty = \frac{\mathcal{E}}{R}$



• find $i(t)$?

$$\sum V_{a..a} = 0$$

$$\mathcal{E} - L\frac{di}{dt} - Ri = 0 \Rightarrow \mathcal{E} = L\frac{di}{dt} + Ri$$

Solution:
$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

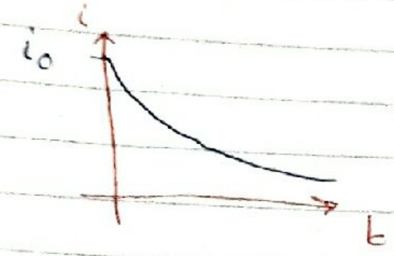
$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right), \quad \tau_L = \frac{L}{R} \text{ sec}$$

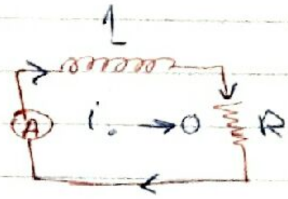
$$V_R(t) = Ri$$

$$V_R = \mathcal{E}\left(1 - e^{-t/\tau_L}\right)$$

$$V_L = \mathcal{E}_L = L\frac{di}{dt} = \mathcal{E} e^{-t/\tau_L}$$

⇒ Removing ε from the Circuit:





$$0 = L\frac{di}{dt} + Ri$$

$$i(t) = i_0\, e^{-Rt/L}$$

$$i(t) = i_0\, e^{-t/\tau_L}$$

⇒ Energy stored in a magnetic field:

$$\left(\varepsilon = Ri + L\frac{di}{dt}\right) i$$

$$\varepsilon i = Ri^2 + iL\frac{di}{dt}$$

$P_\varepsilon = \varepsilon i$

$P_R = i^2 R$ watt

$\frac{dU_B}{dt} = Li\frac{di}{dt}$ watt

$$\int dU_B = \int_0^{i_{max}} Li\,di$$

$$U_B = \frac{1}{2} L i^2 \text{ Joul}$$

⇒ Energy density of magnetic field:

$$u_B = \frac{U_B}{\text{Volume}} = \frac{1}{2}\frac{B^2}{\mu_0} \quad \text{Joul } m^3$$

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \quad J/m^3$$